

FAR WESTERN UNIVERSITY
Semester End Examination-2081
Calculus and Analytical Geometry (CSIT.113)

Faculty: Science and Technology (CSIT)

Level: Undergraduate

Semester: First

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Full Marks: 100

Time: 3hrs.

Group – 'A'

Attempt all questions (Very short answer questions)

8×3 = 24

1. Show that the series $\sum_{n=0}^{\infty} \frac{2(-1)^n}{5 \times 3^n}$ converges to $\frac{3}{10}$
2. Find the equation of the parabola whose vertex is at (-1,1) and whose directrix is $y = 3$
3. Find the angle between the vectors
 $\vec{u} = 4\vec{i} - 2\vec{j} - \vec{k}$ and $\vec{v} = 4\vec{i} - 2\vec{j} + 4\vec{k}$
4. Find volume of the parallelepiped determined by vectors
 $\vec{u} = \vec{i} + \vec{j} - \vec{k}$, $\vec{v} = -2\vec{i} + 3\vec{k}$ and $\vec{w} = 7\vec{j} - 4\vec{k}$
5. Find the n^{th} derivative of $\frac{1}{x^2 - a^2}$
6. Find f_x , f_y and f_{xy} of the function $f(x,y) = 3x^2 - 4xy + 2y^2$
7. Show that $\Gamma \frac{9}{2} = \frac{105}{16} \pi$
8. Find the reduction formula of $\int \tan^n x dx$

Group – 'B'

Attempt any five questions. (Short answer questions)

5×8=40

9. State ratio test theorem. Use it to test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$
10. Find the co-ordinates of center, vertices, eccentricity, foci, length of transverse and conjugate axis of the hyperbola $5x^2 - 4y^2 - 20x - 8y = 4$
11. Find the distance from the point S to the line L which passes through P_0 and parallel to \vec{v} and hence find the distance from the point (1, 3, 2) to the line $x = 2 + 2t$, $y = 1 + 6t$, $z = 3$

P.T.O.

12. i. If $u = \log \sqrt{x^2 + y^2 + z^2}$ show that $u_{xx} + u_{yy} + u_{zz} = \frac{1}{x^2 + y^2 + z^2}$

ii. If $V = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ show that $V_{xx} + V_{yy} + V_{zz} = 0$

13. Find the reduction formula of $\int \cos^m x \sin x dx$

14. Integrate the followings

i. $\int_{\pi}^{2\pi} \int_0^{\pi} (\sin x + \cos y) dx dy$

ii. $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dz dy dx$

Group – 'C'

Attempt any three questions. (Long answer questions)

3×12 = 36

15. a. Give the statement of the integral test. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$ by integral test.
- b. Give the statement of Cauchy n^{th} root test. Examine the convergence of the series $\sum \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^2}$
16. Apply gamma function prove the followings
 - i. $\int_0^{\pi/2} \sin^4 x \cos^2 x dx = \frac{\pi}{32}$
 - ii. $\int_0^1 x^6 \sqrt{1 - x^2} dx = \frac{5\pi}{256}$
 - iii. Use property of definite integral prove that $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{1 + \sqrt{\cot x}} dx = \frac{\pi}{4}$
17. if $\vec{v} = 2\vec{i} + 10\vec{j} - 11\vec{k}$ and $\vec{u} = 2\vec{i} + 2\vec{j} + \vec{k}$ find the followings
 - i. $\vec{u} \cdot \vec{v}$
 - ii. $\vec{u} \times \vec{v}$
 - iii. the cosine of angle between \vec{u} and \vec{v}
 - iv. the vector projection of \vec{u} on \vec{v}
18. Give the statement of the Leibnitz's theorem for n^{th} derivative of product of two functions. If $y = e^{a \tan^{-1} x}$, prove that
 - i. $(1 + x^2)y_2 + (2x - a)y_1 = 0$
 - ii. $(1 + x^2)y_{n+2} + (2nx + 2x - a)y_{n+1} + n(n+1)y_n = 0$
